HOW TO MAXIMAZE THE SUNLIGHT

ETHAN PAN

Multivariable Calculus

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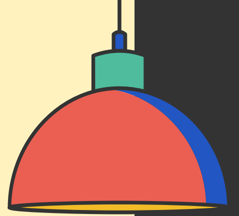
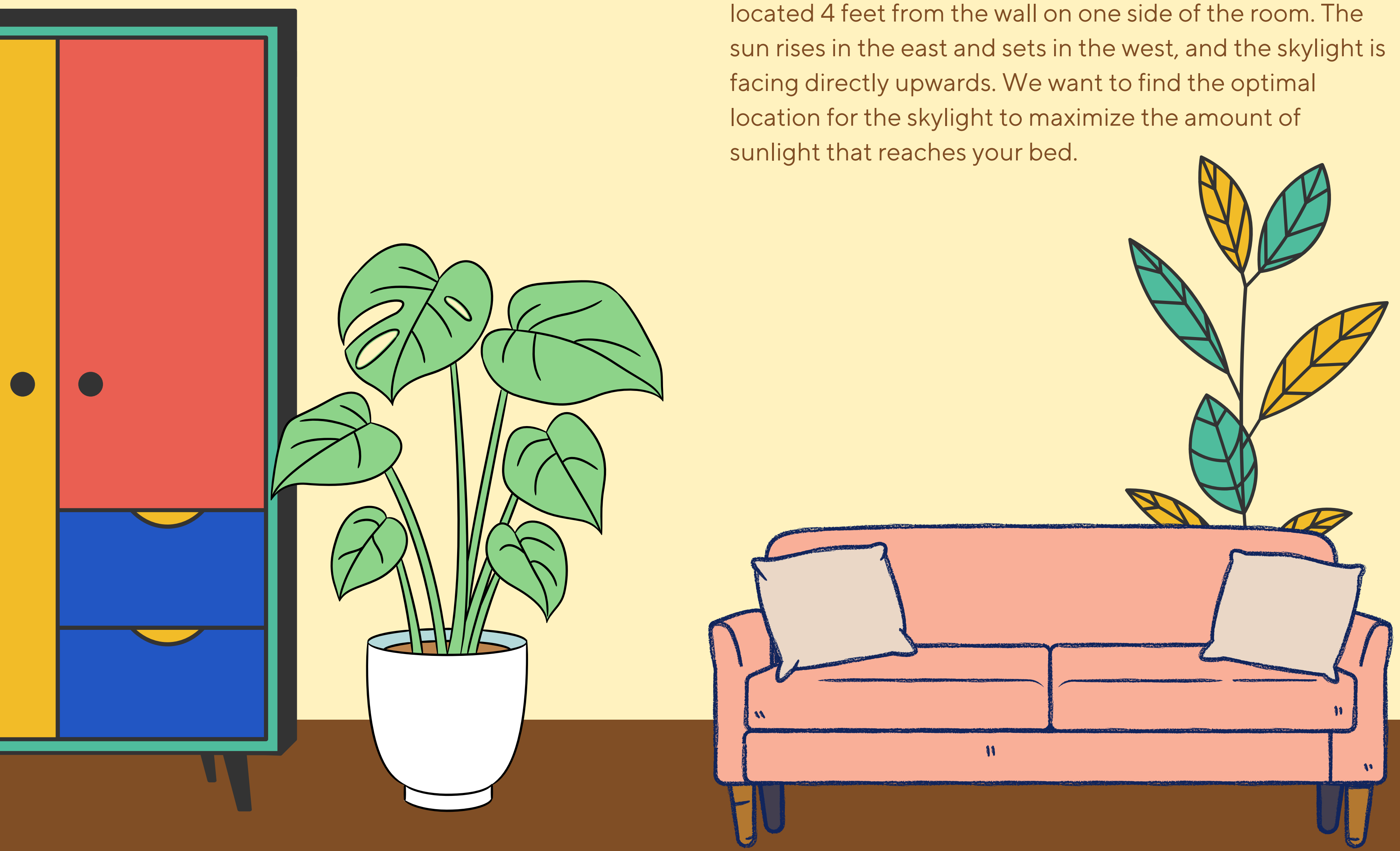
#1 QUESTIONS



Have you ever woken up to a dark and gloomy morning, wishing you could have more sunlight in your room? In this project, we will use multivariable calculus to determine the optimal placement of a skylight in your room to maximize the amount of sunlight that passes through it and reaches your bed in the morning.

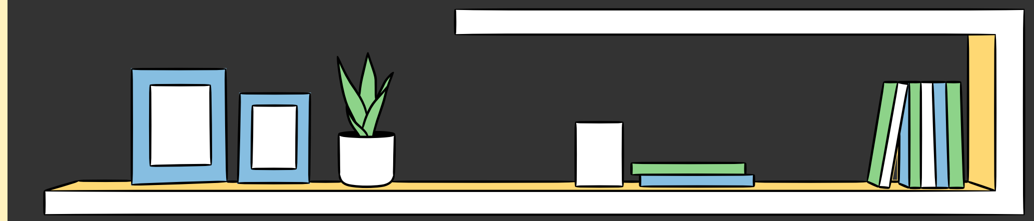
#2 EXPERIMENT DESIGN

Suppose your room is 10 feet wide, 12 feet long, and 8 feet high. You want to place a skylight in the ceiling to maximize the amount of sunlight that reaches your bed in the morning. The skylight has a diameter of 2 feet and is located 4 feet from the wall on one side of the room. The sun rises in the east and sets in the west, and the skylight is facing directly upwards. We want to find the optimal location for the skylight to maximize the amount of sunlight that reaches your bed.



3 SOLUTION

Suppose your room is 10 feet wide, 12 feet long, and 8 feet high. You want to place a skylight in the ceiling to maximize the amount of sunlight that reaches your bed in the morning. The skylight has a diameter of 2 feet and is located 4 feet from the wall on one side of the room. The sun rises in the east and sets in the west, and the skylight is facing directly upwards. We want to find the optimal location for the skylight to maximize the amount of sunlight that reaches your bed.



$$\frac{\partial I}{\partial x} + \frac{\partial I}{\partial y} + \frac{\partial I}{\partial z} = \frac{S}{r^2}\cos(\theta)\cos(\phi)$$

In this equation, *I* represent the **intensity** of the sunlight, *x*, *y*, and *z* represent the **coordinates of the skylight**, *S* represents the **solar constant** (the amount of energy that the sun emits per unit area), *r* represents the **distance** between the skylight and the sun, *θ* represents the **angle** between the sun and the skylight, and *φ* represents the **angle** between the skylight and the ceiling.

First, we need to calculate the distance between the skylight and the sun. We can use the Pythagorean theorem to find the distance:

$$r = \sqrt{(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2}$$

where (x_s, y_s, z_s) represents the position of the sun.

In this case, we know that the skylight is located 4 feet from the wall on one side of the room, so we can assume that it is located at $(4, 0, 8)$ (since the skylight is facing directly upwards). We also know that the sun rises in the east and sets in the west, so we can assume that it is located at $(0, 0, 0)$ (the origin).

Using these values, we can calculate the distance between the skylight and the sun:

$$r = \sqrt{(4-0)^2 + (0-0)^2 + (8-0)^2} = \sqrt{80} \approx 8.94 \text{ feet}$$

Next, we need to calculate the angles θ and ϕ. We can use trigonometry to find these angles:

$$\theta = \tan^{-1}\left(\frac{y_s - y}{x_s - x}\right)$$

$$\phi = \tan^{-1}\left(\frac{z_s - z}{\sqrt{(x_s - x)^2 + (y_s - y)^2}}\right)$$

Using these values, we can calculate the angles:

$$\theta = \tan^{-1}\left(\frac{0-0}{0-4}\right) = \tan^{-1}(-0) = 0$$

$$\phi = \tan^{-1}\left(\frac{0-8}{\sqrt{(0-4)^2 + (0-0)^2}}\right) = \tan^{-1}(-2) \approx -1.11 \text{ radi}$$

Finally, we can use the equation we derived earlier to calculate the intensity of the sunlight:

$$\frac{\partial I}{\partial x} + \frac{\partial I}{\partial y} + \frac{\partial I}{\partial z} = \frac{S}{r^2}\cos(\theta)\cos(\phi)$$

Plugging in the values we calculated, we get:

$$\frac{\partial I}{\partial x} + \frac{\partial I}{\partial y} + \frac{\partial I}{\partial z} = \frac{1361 \text{ W/m2}}{(\sqrt{80})2}\cos(0)\cos(-1.11) \approx -16.8 \text{ V}$$

This negative value means that the intensity of the sunlight is decreasing as we move in the direction of the skylight. This is because the skylight is not optimally placed to maximize the amount of sunlight that reaches your bed.